4. $A=\begin{bmatrix}5 & -3\\ -4 & 3\end{bmatrix}$, $A-\lambda I=\begin{bmatrix}5-\lambda & -3\\ -4 & 3-\lambda\end{bmatrix}$. The characteristic polynomial of A is

$$\det(A-\lambda I)=(5-\lambda)(3-\lambda)-(-3)(-4)=\lambda^2-8\lambda+3$$

Use the quadratic formula to solve the characteristic equation and find the eigenvalues:

$$\lambda=\frac{8\pm\sqrt{64-4(3)}}{2}=\frac{8\pm 2\sqrt{13}}{2}=4\pm\sqrt{13}$$

8. $A = \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}$, $A - \lambda I = \begin{bmatrix} 7-\lambda & -2 \\ 2 & 3-\lambda \end{bmatrix}$. The characteristic polynomial is

$$\det(A - \lambda I) = (7-\lambda)(3-\lambda) - (-2)(2) = \lambda^2 - 10\lambda + 25$$

Since $\lambda^2 - 10\lambda + 25 = (\lambda - 5)^2$, the only eigenvalue is 5, with multiplicity 2.

12. Make a cofactor expansion along the third row:

$$\det(A-\lambda I)=\det\begin{bmatrix}-1-\lambda & 0 & 1\\ -3 & 4-\lambda & 1\\ 0 & 0 & 2-\lambda\end{bmatrix}=(2-\lambda)\cdot\det\begin{bmatrix}-1-\lambda & 0\\ -3 & 4-\lambda\end{bmatrix}$$

$$=(2-\lambda)(-1-\lambda)(4-\lambda)=-\lambda^3+5\lambda^2-2\lambda-8$$

14. Make a cofactor expansion along the second row:

$$\det(A-\lambda I)=\det\begin{bmatrix}5-\lambda & -2 & 3\\ 0 & 1-\lambda & 0\\ 6 & 7 & -2-\lambda\end{bmatrix}=(1-\lambda)\cdot\det\begin{bmatrix}5-\lambda & 3\\ 6 & -2-\lambda\end{bmatrix}$$

$$=(1-\lambda)\cdot[(5-\lambda)(-2-\lambda)-3\cdot 6]=(1-\lambda)(\lambda^2-3\lambda-28)$$

$$=-\lambda^3+4\lambda^2+25\lambda-28 \text{ or } (1-\lambda)(\lambda-7)(\lambda+4)$$

5. The determinant of a triangular matrix is the product of its diagonal entries:

$$\det(A-\lambda I)=\det\begin{bmatrix}5-\lambda & 0 & 0 & 0\\ 8 & -4-\lambda & 0 & 0\\ 0 & 7 & 1-\lambda & 0\\ 1 & -5 & 2 & 1-\lambda\end{bmatrix}=(5-\lambda)(-4-\lambda)(1-\lambda)^2$$

The eigenvalues are 5, 1, 1, and −4.

18. Row reduce the augmented matrix for the equation $(A-5I)\mathbf{x}=\mathbf{0}$:

$$\begin{bmatrix} 0 & -2 & 6 & -1 & 0 \\ 0 & -2 & h & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & -2 & 6 & -1 & 0 \\ 0 & 0 & h-6 & 1 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & h-6 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

For a two-dimensional eigenspace, the system above needs two free variables. This happens if and only if $h=6$.

20. $\det(A^T - \lambda I) = \det(A^T - \lambda I^T)$

$= \det(A - \lambda I)^T$ Transpose property

$= \det(A - \lambda I)$ Theorem 3(c)